

Appendix A

Matrix Green function

The matrix Green function $\mathbf{G}(z, \xi)$ introduced in (5.41) is given by

$$\mathbf{G}(z, \xi) = \begin{pmatrix} G_{11}(z, \xi) & 0 \\ 0 & G_{22}(z, \xi) \end{pmatrix}$$

where

$$G_{11}(z, \xi) = \begin{cases} g_{11}(z, \xi) & \text{if } z \leq \xi \\ g_{11}(z, \xi) + \frac{a \sinh \alpha(z - \xi) - \alpha \sinh a(z - \xi)}{a\alpha\tilde{D}aR_m(\alpha^2 - a^2)} & \text{if } z > \xi, \end{cases}$$

$$G_{22}(z, \xi) = \begin{cases} g_{22}(z, \xi) & \text{if } z \leq \xi \\ g_{22}(z, \xi) + \frac{a(z - \xi) \cosh a(z - \xi) - \sinh a(z - \xi)}{2\tilde{D}aP_m a^3} & \text{if } z > \xi, \end{cases}$$

$$\text{with } \alpha = \sqrt{\frac{1 + a^2\tilde{D}a}{\tilde{D}a}},$$

$$\begin{aligned} g_{11}(z, \xi) &= \frac{\alpha \sinh a(1 - \xi) - a \sinh \alpha(1 - \xi)}{a\alpha\tilde{D}aR_m(\alpha^2 - a^2)} \\ &\cdot \left\{ \frac{a\alpha[\cosh a(1 - z) - \cosh 2\alpha \cosh a(z + 1)] + \alpha^2 \sinh 2\alpha \sinh a(z + 1)}{2a\alpha(1 - \cosh 2\alpha \cosh 2a) + (a^2 + \alpha^2) \sinh 2\alpha \sinh 2a} \right. \\ &+ \frac{a\alpha[\cosh \alpha(1 - z) - \cosh 2a \cosh \alpha(z + 1)] + a^2 \sinh 2a \sinh \alpha(z + 1)}{2a\alpha(1 - \cosh 2\alpha \cosh 2a) + (a^2 + \alpha^2) \sinh 2\alpha \sinh 2a} \Big\} \\ &+ \frac{\cosh a(1 - \xi) - \cosh \alpha(1 - \xi)}{\tilde{D}aR_m(\alpha^2 - a^2)} \\ &\cdot \left\{ \frac{a \sinh 2\alpha \cosh a(z + 1) - \alpha[\sinh a(1 - z) + \cosh 2\alpha \sinh a(z + 1)]}{2a\alpha(1 - \cosh 2\alpha \cosh 2a) + (a^2 + \alpha^2) \sinh 2\alpha \sinh 2a} \right. \\ &+ \frac{\alpha \sinh 2a \cosh \alpha(z + 1) - a[\sinh \alpha(1 - z) + \cosh 2a \sinh \alpha(z + 1)]}{2a\alpha(1 - \cosh 2\alpha \cosh 2a) + (a^2 + \alpha^2) \sinh 2\alpha \sinh 2a} \Big\} \end{aligned}$$

and

$$\begin{aligned}
g_{22}(z, \xi) = & \frac{\sinh a(1 - \xi) - a(1 - \xi) \cosh a(1 - \xi)}{2\tilde{D}aP_ma^3(4a^2 - \sinh^2 2a)} \{a^2 \cosh a(1 - z) \\
& - [(a^2 + 1) \sinh 2a + 2a \cosh 2a] \sinh a(z + 1) \\
& + a(\sinh 2a + a \cosh 2a) \cosh a(z + 1)\} \\
& + \frac{(1 - \xi) \sinh a(1 - \xi)}{2\tilde{D}aP_ma(4a^2 - \sinh^2 2a)} \{a \sinh a(1 - z) \\
& - (\sinh 2a + a \cosh 2a) \sinh a(z + 1) + a \sinh 2a \cosh a(z + 1)\} \\
& + z \frac{\sinh a(1 - \xi) - a(1 - \xi) \cosh a(1 - \xi)}{2\tilde{D}aP_ma^2(4a^2 - \sinh^2 2a)} \{a \cosh a(1 - z) \\
& + \sinh a(1 - z) \\
& + (\cosh 2a - a \sinh 2a) \sinh a(z + 1) + a \cosh 2a \cosh a(z + 1)\} \\
& + z \frac{(1 - \xi) \sinh a(1 - \xi)}{2\tilde{D}aP_ma(4a^2 - \sinh^2 2a)} \{a \sinh a(1 - z) \\
& + (\sinh 2a - a \cosh 2a) \sinh a(z + 1) + a \sinh 2a \cosh a(z + 1)\}.
\end{aligned}$$